Monte Carlo Investigation of Thrust Imbalance of Solid Rocket Motor Pairs

Richard H. Sforzini* and Winfred A. Foster Jr.†
Auburn University, Auburn, Ala.

The Monte Carlo method of statistical analysis is used to investigate the theoretical thrust imbalance of pairs of solid rocket motors (SRMs) firing in parallel. Sets of the significant variables are selected using a random sampling technique and the imbalance calculated for a large number of motor pairs using a simplified, but comprehensive, model of the internal ballistics. The treatment of burning surface geometry allows for the variations in the ovality and alignment of the motor case and mandrel as well as those arising from differences in the basic size dimensions and propellant properties. The analysis is used to predict the thrust-time characteristics of 130 randomly selected pairs of Titan IIIC SRMs. A statistical comparison of the results with test data for 20 pairs shows the theory underpredicts the standard deviation in maximum thrust imbalance by 20% with variability in burning times matched within 2%. The range in thrust imbalance of Space Shuttle type SRM pairs is also estimated using applicable tolerances and variabilities and a correction factor based on the Titan IIIC analysis.

Introduction

THE principal objective of the research was to develop a technique for statistically investigating the thrust imbalance of a pair of solid rocket motors (SRMs) firing in parallel. The study of thrust balance and imbalance is of particular interest for application to the NASA Space Shuttle because two very large SRMs fire in parallel on the Shuttle. A similar arrangement was utilized in the Titan program with somewhat smaller motors, but the differences between the Titan and Shuttle make it more imperative that the thrust on the Shuttle SRMs be balanced to assure proper guidance and control of the vehicle.

Past analyses (e.g., Ref. 1) of SRM reproducibility have been concerned mainly with characteristics of a population of single motors rather than of pairs. Usually a non-time dependent parameter such as average thrust or total impulse has been of interest. The distributions of variables affecting the parameter were assumed normal, in the statistical sense, and cross-correlation effects were neglected.

For the present investigation, the Monte Carlo technique[2] was selected. This method is not limited to normal distributions of the input variables. Errors arising from neglecting the cross-correlation of variables are minimized by selecting for the most part completely independent input variables. The large number of such variables makes recourse to the Monte Carlo method advantageous because of the prohibitive amount of analysis a parametric approach would require. Also, the Monte Carlo approach is more realistic than simply evaluating the worst-case conditions. The accuracy of the predictions will, of course, depend to a large extent on the availability of data to define accurately the statistical distributions of input variables.

Performance Model

The basic computer program for the internal ballistics presented in Refs. 3-5 is used for performance simulation. The internal ballistics analysis includes representation of mass addition, erosive burning and throat erosion. It is assumed that the propellant does not break up and is extinguished only by being completely consumed by burning normal to the propellant surface. The burning surface is assumed to regress linearly along the grain length consistent with the differences in surface regression at the head and aft ends, for which burning rates are calculated separately. Support for this assumption as an approximation may be found in the results of interrupted burning tests.[6] Separate determination of head and nozzle end burning rates provides a means of accounting for one of the principal effects of a surface regression which varies along the length of the grain. This is the effect on tailoff characteristics which, in somewhat simplified terms, is treated by reducing the burning surface at tailoff in proportion to the length of propellant that has experienced burnout, according to the linear model.

Table 1 outlines the statistical variables which are used for a segmented SRM with a combination circular perforated and star grain. The majority of these variables are standard design dimensions and propellant characteristics. Several of the inputs which necessitated revision to the analyf Refs. 3-5 are discussed further.

A circular perforated grain exterior or interior perimeter is not perfectly round. The ovality and any lack of concentricity of the grain perforation with respect to the motor case can clearly influence the ballistic performance of SRMs during the

Table 1 Statistical input variables for segmented SRM with combination circular perforated and star grain

Component	Characteristics	Number of variables
Propellant	density	1
	bulk temperature	1
	burning rate	6[a]
	ignition delay	1
	composition	1
Nozzle	initial dimensions	4
	throat erosion rate	1[b]
Circular perforated grain	basic dimensions	11
	ovality	6
	concentricity	4
Star grain	basic dimensions	5
		41

[a]Burning rate coefficients and exponents above and below an input transition pressure and 2 coefficients for the Robillard-Lenoir erosive burning rate equation.[7] [b]Systematic variations due to changes in nozzle throat pressure are also included in the evaluation.

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*Professor, Aerospace Engineering. Associate Fellow, AIAA.
†Assistant Professor, Aerospace Engineering. Member AIAA.

critical tailoff period. The burning surface geometry is defined by one fore and one aft radial reference plane. The reference planes must intersect the cylindrical portion of the rocket motor case to eliminate the effects of end closures on the geometric properties to be calculated. The implied assumption is that the cylindrical portion of the rocket dominates the influences of ovality and misalignment. This is valid for motors with high L/D, e.g., the Titan IIIC and the Space Shuttle SRMs.

The geometry of the reference planes is illustrated in Fig. 1. Both the exterior and interior (bore) perimeters of the grain are assumed to be distorted into elliptical perimeters from nominally circular ones. With some restrictions and loss in accuracy, the analysis is also applicable to grains with star-shaped perforations. However, when both circular perforated and star grains are present in the same motor as in the sample cases of this article, only the former is used to determine the effects of ovality and misalignment. The exterior and interior grain perimeters are assumed to remain elliptical as burning progresses which is not rigorous for burning normal to the surface, but the error introduced is insignificant for the small degrees of ovality encountered in practice.

Burning perimeters for each reference plane vary from one SRM to another as a result of variability in the ovality (out-of-roundness) of both the grain bore and exterior, the angular orientation α of the ovalities, and the eccentricity of the grain bore with respect to its exterior. The latter is specified by the independent variables e_x and e_y. It would appear that the head end and aft end geometric features are more or less independent and they are treated as such in the remainder of this article. The indjependent specification of e_x and e_y at both ends of the rocket provides for a statistically distributed misalignment of the grain bore and exterior.

Using the model described above, geometric correction factors are calculated and applied to standard calculations of perimeter. When the ovality model indicates burn-throughs have occurred at adjacent reference planes, the burning perimeters are assumed to vary linearly between planes. When burn-through has occurred at only one of two adjacent reference planes, that portion of the correction factor applicable to each end is weighted in proportion to the corresponding length that has or has not experienced burn-through.

A separate analysis[8] of the effects of all the input variables shows that each misalignment and ovality variable by itself makes only a small contribution to thrust imbalance. However, when coupled with each other and with the additional variables on a statistical basis the misalignment and ovality variables can be quite significant.

Characteristic velocity and ratio of specific heats, C^* and γ, respectively, are calculated based on $R_{0A\ell}$, the ratio of the oxidizer to aluminum of the propellant. In this way the cross-correlation between C^* and γ are incorporated. Data for determining separate relationships for C^* and γ as functions of $R_{0A\ell}$ are obtained from thermochemical analysis with no allowance for inefficiency. Because of the relatively strong influence of the aluminum on the propellant chemistry the two-component oxidizer to aluminum system should convey the largest effects of composition on C^* and γ. Systematic variations have also been incorporated to give the effects of chamber pressure and propellant bulk temperature on C^* and of chamber pressure on γ.

The introduction of $R_{0A\ell}$ as an independent variable raises the question of possible cross-correlation with other independent propellant variables, especially the burning rate coefficient a and density ρ. When such correlations can be identified they may of course be treated in a manner similar to that given for C^* and γ. Although the matter has not been investigated fully, it appears that factors such as oxidizer particle size distributions, amount of burning rate catalyst and other composition variables will play a more important role than the oxidizer to aluminum ratio in fixing the *distributions*

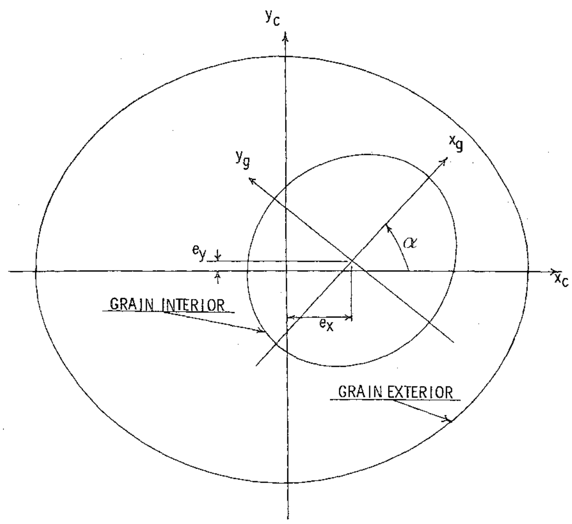

Fig. 1 Geometry for analysis of ovality and misalignment.

of a and ρ. In any event, cross-correlation between $R_{0A\ell}$ and ρ is assumed negligible as has, with somewhat less justification, the correlation between a and ρ. The latter warrants further investigation as it is intuitively clear that at least a weak cross-correlation exists between a and ρ because of the effects of oxidizer particle size distribution on these parameters.

An ignition delay time has been incorporated as a statistical input variable. When properly specified the delay time reflects the effects of spatial variability in ignition initiation and variation in characteristic chamber length. Systematic variations in delay time due to grain temperature variations are calculated by the program. Also, the initial equilibrium pressure (at the end of the delay time) depends on the variation in input propellant properties and initial grain geometry.

A more rigorous model of the internal ballistics might be adopted, but a more cumbersome application of the Monte Carlo technique would result. We believe the simplified model is consistent with the degree of accuracy with which input data is generally known. Also, differences in performance within pairs of SRMs is the item of interest and biases introduced by the approximations tend to reflect equally in each motor of a pair. Biases in the model should have a similar effect on the distribution of a performance difference as that of shifts in the distributions of input variables. To obtain an indication of the magnitude of the effect, the mean values of the most significant variables[8] were allowed to change between each of 50 SRM pairs. This was accomplished by assigning standard deviations (σ's) of burning rate coefficient and propellant density *means* between pairs that were three and two orders of magnitude higher, respectively, then the σ's assigned to within-pair variations. Nozzle throat erosion rate and $R_{0A\ell}$ means were varied to a lesser extent. The within-pair σ's of all variables were maintained the same between two evaluations, one with and one without the large σ's in the means of the four variables mentioned. The character of the two distributions of maximum thrust imbalance, as calculated by the Monte Carlo analysis to be described, was essentially the same with the σ's differing by only 1.5%.

Input Variables

The Monte Carlo approach[2] randomly selects numbers from zero to one and relates these numbers to the values of input variables through the distribution functions that have been specified for each variable. Establishing reasonable distributions is the most difficult part of the study because of

Table 2 Means (μ) and standard deviations (σ) of input variables for the sample cases[a]

Component/variable	Units	Titan IIIC μ	Titan IIIC σ	Space Shuttle type μ	Space Shuttle type σ
Propellant					
density	lbm/in^3	0.0630	1.00×10^{-5}	0.0635	1.05×10^{-5}
bulk temperature	°F	80.0	0.1833	60.0	0.2333
rate coefficient	in./sec-psi^n	0.0665	3.428×10^{-4}	0.0366	2.19×10^{-5}
ignition delay	msec	237	9.08	400	15.3[b]
oxidizer wt/Al wt	1	4.250	0.04	4.350	0.04
Nozzle					
throat diam	in.	37.70	0.0333	54.430	0.0100
exit diam	in.	106.63	0.0333	145.67	0.03333
throat erosion rate	mils/sec	4.67	0.262	7.63[c]	0.320[c]
exit half angle	deg	11.25	0.0833	11.25	0.0
cant angle	deg	0.0	0.833	0.0	0.0
Circular perforated grain					
length mean outside diam	in.	119.98	0.01462	143.08	0.01462
length mean inside diam	in.	47.60	0.03333	63.59	0.03333
main grain length with	in.	613.10	0.7453	1135.58	0.5770
inside radial taper	in.	5.00	0.01054	2.41	0.02357
outside radial taper	in.	0.0	0.02357	0.0	0.02357
aft tapered length with	in.	0.0	0.0[d]	176.5	0.0[d]
inside radial taper	in.	0.0	0.0	3.040	0.02357
4 radial out-of-rounds	in.	0.0	0.08333	0.0	0.08333
4 concentricities	in.	0.0	0.050	0.0	0.050
2 ovality orientations	deg	0.0	random	0.0	random
Star grain					
grain length	in.	33.0[e]	0.1667	189.15[e]	0.3333
outside radius	in.	59.988	0.00731	71.540	0.00731
fillet radii	in.	3.0	0.01179	2.010	0.01111
web radius	in.	50.0	0.01667	63.54	0.01667

[a] A few of the least important variables have been omitted. [b] Same coefficient of variation as Titan IIIC. [c] Data based on Poseidon program. [d] The effect of variations in the aft tapered length is negligible. [e] Head end geometry is represented by tabular (nonstatistical) values.

the lack of reliable statistical data on all of the numerous variables. Even when large populations exist from which data may be directly applied or scaled to another SRM, documentation of manufacturing variations is often incomplete.

Table 2 gives the input population means μ and standard deviations σ of the statistical variables used for the theoretical sample cases to be examined. Although variabilities of a number of input variables shown in Table 2 were specified by other than normal distributions, they were reasonably close to normal so that specification of the μ and σ should suffice for concise description of input. Selection of the statistical distributions for some of the inputs was necessarily somewhat arbitrary although state-of-the-art values have been used. For the Titan IIIC specific data was available on the distribution of bulk grain temperature, ignition delay and throat erosion rate. For dimensional variables, the convention is adopted of taking standard drawing tolerances as representing $\pm 3\sigma$ in a normally distributed population of a variable. Also, where more than one dimension controls a variable input dimension, the σ of the variable is taken as the square root of the sum of the squares of the σ's of the controlling variables, assumed to be normally distributed and uncorrelated. An example of this is the σ of the average outside diameter of the circular perforated grain which is calculated based on the σ's of the outside diameter of the case, and the thicknesses of the case wall, liner and insulation. The way a particular variable is used in the program must also be considered. Thus, when a dimension (or other characteristic) is subject to random variation and the average variation is required by the program, the σ in the variable is reduced. For example, the σ of the fillet radii of the star points is divided by the square root of the number of star points because each star point has an equal effect on the burning surface. Similarly, the real propellant average burning rate variation within pairs may be reduced substantially by the method of propellant selection and division of propellant from several mixers between a pair of SRMs. Special attention must be given to the determination of the burning rate coefficient distribution in each situation.

Sample Cases

Where large SRMs have been produced for firing in pairs the Titan IIIC program offers a singularly good potential source of both input data and performance results to form a basis for examination of the efficacy of the Monte Carlo analysis. Unfortunately, a vital element of data, the distribution of the burning rate coefficient of the propellant, has not been available. Variations in the burning rate coefficient may account for the majority of the variations in web action time for the ordinery SRM and hence in thrust imbalance for a pair of SRMs firing in parallel.[8] The burning rate coefficient data was extracted from the test data on web action time using the Monte Carlo program.

The statistical characteristics of all distributed input variables except burning rate coefficients were assigned from Table 2 as previously discussed. The burning rate coefficient was assumed to be normally distributed, but the value of its standard deviation was somewhat arbitrarily selected. After several attempts with different standard deviations for the burning rate coefficient, a value of the coefficient was found for which the distribution of burning times obtained from the theoretical analysis matched closely that determined from test data. Naturally, no matter how poor the theoretical analysis, such a value can be found. However, using the value of burning rate coefficient thus determined, the statistical program also compares well with test data with regard to the distribution of maximum thrust imbalance. The correspondence would appear to be more than fortuitous and tends to validate the analysis.

For the purpose of obtaining the match in burning time distributions, the standard deviation of the burning rate coefficient was adjusted so that the computed average of the second moments about zero (s_0) of the differences in action times and the differences in web action times of the pairs matched the corresponding average from the test data to within 2%. The second moment about zero was used rather than the standard deviation because the time differences are all recorded as positive. The average values of the moments for the two burning times were used to minimize the effects of possible inconsistencies or biases in the determination of the times. For example, web action time is obtained from actual performance data by the two-tangent angle bisection method while the computer program determines the time at which the first burn-through of the main propellant web would occur in the absence of misalignment and ovality of the grain.

Figure 2 shows histograms of the thrust imbalances for the theoretical assessment of 130 Titan IIIC pairs and for the actual performance of 20 Titan IIIC pairs. While the theoretical s_0 differed by 20% from the test data, the agreement is judged reasonably good. The theoretical model should, of course, underestimate the thrust imbalance as not all contributing factors have been included. It is noteworthy that the maximum value of thrust imbalance calculated for the 130 SRM pairs was 160,550 lbf while the maximum observed value for 20 pairs was 157,000 lbf. A meaningful quantitative comparison of the time at which the maximum thrust imbalance occurs is difficult because this time is clearly subject to wide variations among those pairs for which the imbalance is low and therefore relatively insignificant. However, for those pairs for which the absolute value of maximum thrust imbalance is above the mean, the maximum occurs within 0.5 sec after the second SRM begins tailoff. A statistical analysis of the Titan performance data indicates the highest values of thrust imbalance are anticipated in the region of 1.5 to 3.0 sec after the second motor begins its (15 sec) tailoff.[9] This disparity must again be attributed largely to the limitations of the performance model.

A first estimate is now made of the statistical performance of pairs of 146-in. dia. SRMs of the type to be used on the Space Shuttle. Thirty-seven mixes of propellant are to be used to cast a single center segment so that the σ of average burning rate should be reduced from that ($3\sigma/\mu = 4.3\%$) usually obtained when SRMs are cast from single mixes. Also, alternating mixes from the two mixers to be used between SRMs of

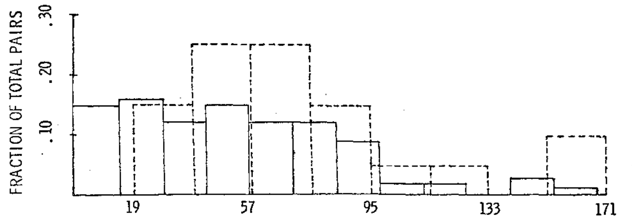

Fig. 2 Histograms of absolute values of maximum thrust imbalance for Titan IIIC SRM pairs.----20 pairs (test) $s_0 = 83,180$.——130 pairs (theory) $s_0 = 66,420$.

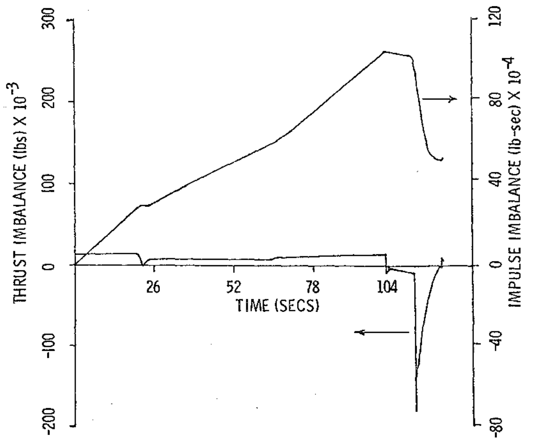

Fig. 3 Typical thrust and impulse imbalances for one pair of 146-in. diam SRMs.

a pair will further reduce burning rate variation. Considering these factors, statistical analysis indicates $3\sigma/\mu = 0.18\%$ for the average burning rate between motors of a pair.

Figure 3 shows the theoretical time-varying performance of one pair of SRMs and Table 3 gives a portion of the evaluation of 50 SRM pairs. To obtain a specific estimate of the maximum thrust imbalance to be anticipated, $\bar{X} \pm Ks$ for the sample distribution of the thrust imbalances is examined. Here $\bar{X}$ and s are the mean and standard deviation of the sample, respectively, and K is the confidence coefficient for two-sided tolerance limits.[10] The coefficient K is selected such that the probability is 90% that at least 99.9% of the total population will be within the limits of $\bar{X} \pm Ks$. The confidence coefficient used (3.833) applies only to a normally distributed total population. It is assumed that the distribution of the absolute values of thrust imbalance is the upper half of a normal distribution of algebraic values of thrust imbalance with $\bar{X} = 0$. For the distribution of algebraic values $s^2 = \bar{X}^2_{\mathrm{abs}} + s^2_{\mathrm{abs}}$, where abs denotes the absolute values of thrust imbalance, and the calculated limits are $\pm 483{,}500$ lbf. If the assumption is made that the s calculated for the Space Shuttle pairs is in error by the same percentages as the s_0 calculated for the Titan deviates from test results, the predicted limits for the larger pairs are $\pm 580{,}200$ lbf.

The applicability of tolerance limits based on a normally distributed population has not been firmly established. Indeed, chi-square tests of the sample distributions of the maximum thrust imbalances indicate rather low probabilities of normality for both the theoretical and test samples. Methods also exist for establishing tolerance limits without any assumption about the form of the distribution, but the limits are obviously broader than those for a normal population[10] and may be ultra-conservative unless the sample size is very large.

The limits calculated for the maximum thrust imbalance are about 3/4 to 1/2 those calculated by various methods of scaling Titan IIIC data to the Space Shuttle using factors

Table 3 Means ($\bar{X}$) and standard deviations (s) of selected performance characteristics for fifty 146-in. diam SRMs

	$\bar{X}$	s
Absolute value of maximum thrust imbalance during web action time (AFMAX)lbf	19,620	9250
Time of AFMAX, sec	83.89	36.59
Absolute value of maximum thrust imbalance during tailoff (AFMAXT) lbf	110,346	61,130
Time of AFMAXT, sec	111.60	0.93
Absolute value of the difference in time at which the two motors of a pair begin tailoff, sec	0.20	0.14
Absolute value of the thrust imbalance at input time of maximum dynamic pressure, lbf	2954	3966
Algebraic value of the impulse imbalance during tailoff, lbf-sec	−51,060	461,800
Absolute value of the area between the thrust-time traces of the pair during tailoff, lbf-sec	406,400	237,500
Absolute value of thrust imbalance when last motor of pair reaches 100,000 lb. thrust during tailoff (DF100K) lbf	8,555	13,470
Time of DF100K, sec	118.66	0.29

which involve only the ratios of the thrusts and total tailoff times for the two different motors and assuming a normally distributed population to establish tolerance limits. We believe such scaling to be inaccurate because it generally does not consider the possibility of different controls over the input variables.

Conclusions

The technique described gives a method for predicting variations in the performance of pairs of SRMs on a statistical probability basis. Comparisons of the theoretical approach with actual test data show reasonably good agreement for Titan IIIC SRMs. For other SRMs, the accuracy of predictions based on this method will depend to a large extent on the availability of specific data to define accurately the statistical distributions of the input variables. One value of this study may be that of showing the need for acquisition of detailed statistical data on manufacturing and processing variables.

Aside from providing a technique for direct theoretical prediction of performance variation, the Monte Carlo method provides an approach to defining the quality of the various statistical distributions of performance differences of interest based on as large a number of SRMs as desired. The distributions thus obtained may be used in analysis of experimental data to establish confidence coefficients on a more logical basis than simply assuming normal distributions or unknown distributions.

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